Verification of Flat FIFO Systems

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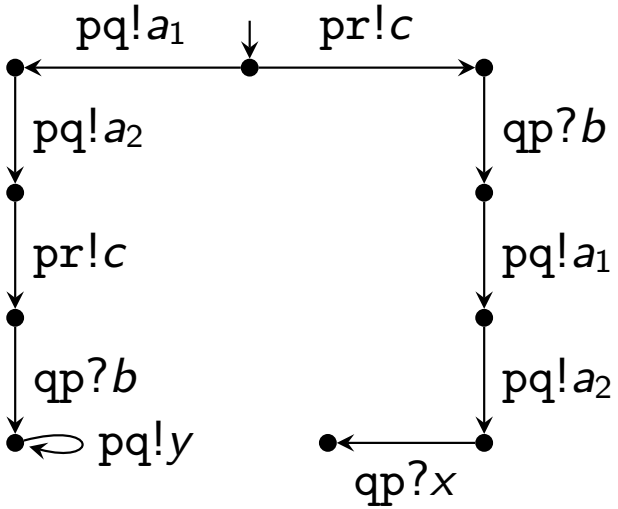
Motivation

Verification of infinite-state FIFO systems

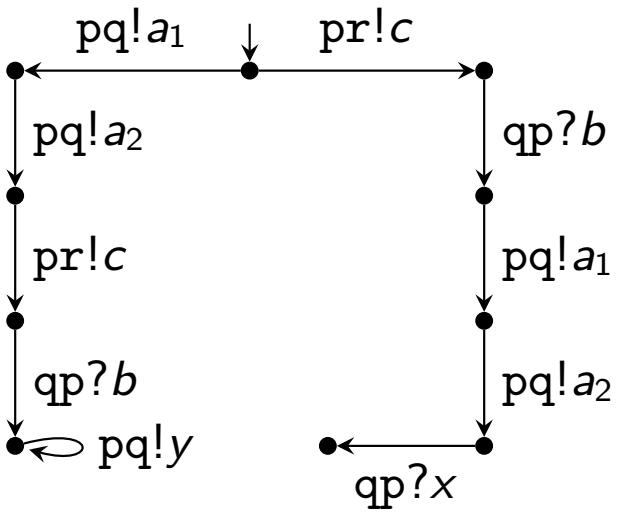
- Model defined in 1970 for communication protocols.
- Difficult to verify since reachability is undecidable.
- Used for choregraphy, contract, interfaces, web services,...
- Reachability is decidable for interesting subclasses.
- Interesting papers about synchronizability (more or less correct).

A FIFO system from (LY 2019) with 3 processes P, Q, R and 4 channels: pq, pr, qp, rq

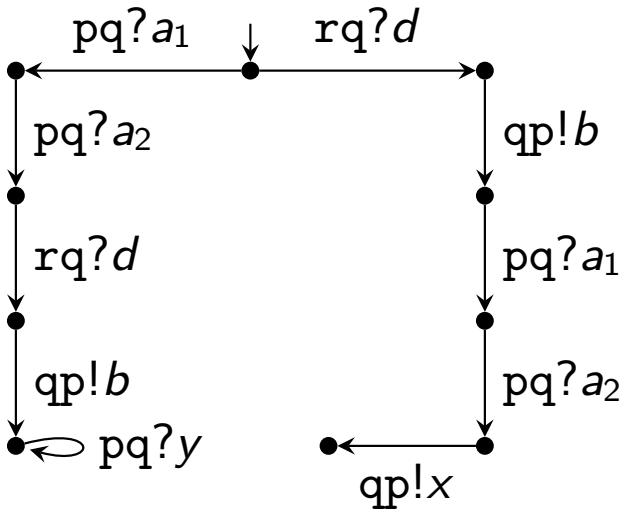


(a) Process P

A FIFO system from (LY 2019) with 3 processes P, Q, R and 4 channels: pq, pr, qp, rq

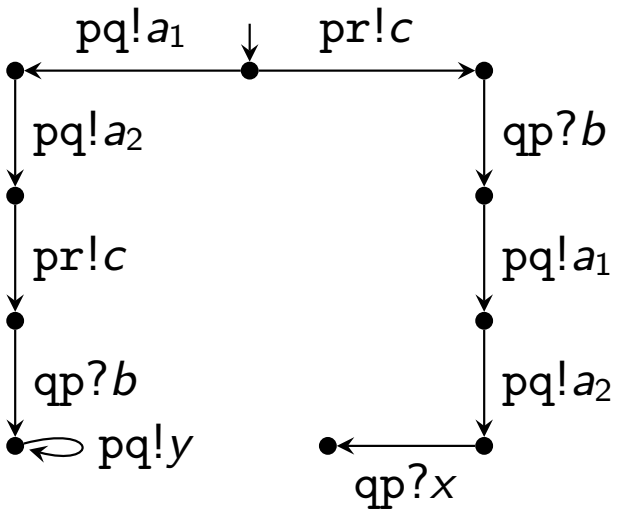


(a) Process P

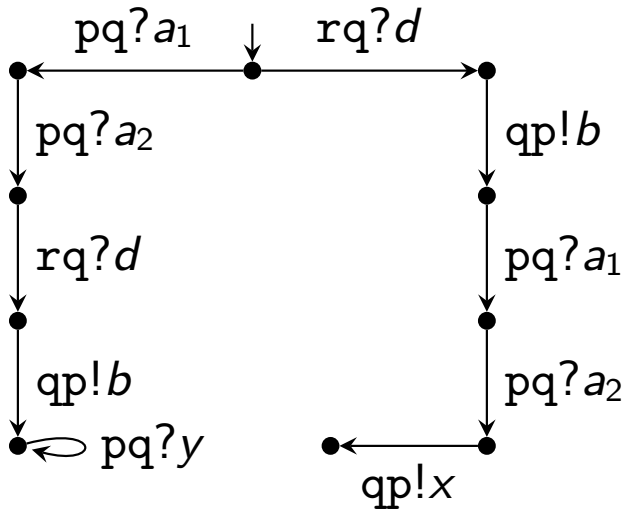


(b) Process Q

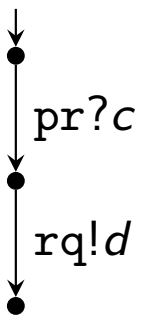
A FIFO system from (LY 2019) with 3 processes P, Q, R and 4 channels: pq, pr, qp, rq



(a) Process P

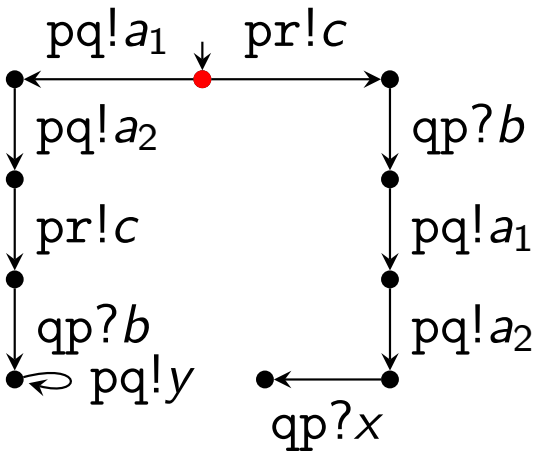


(b) Process Q

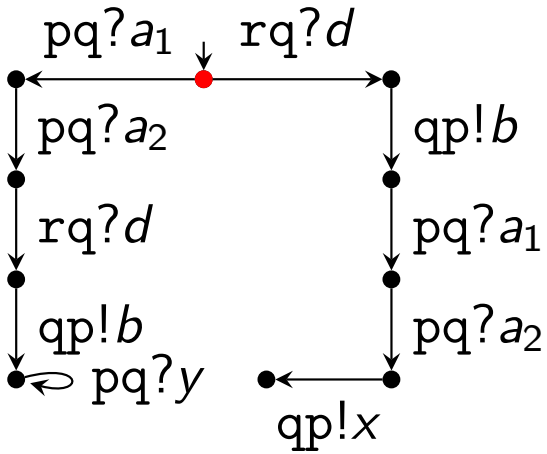


(c) Process R

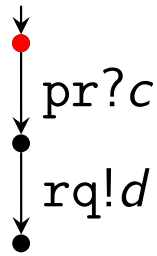
process P

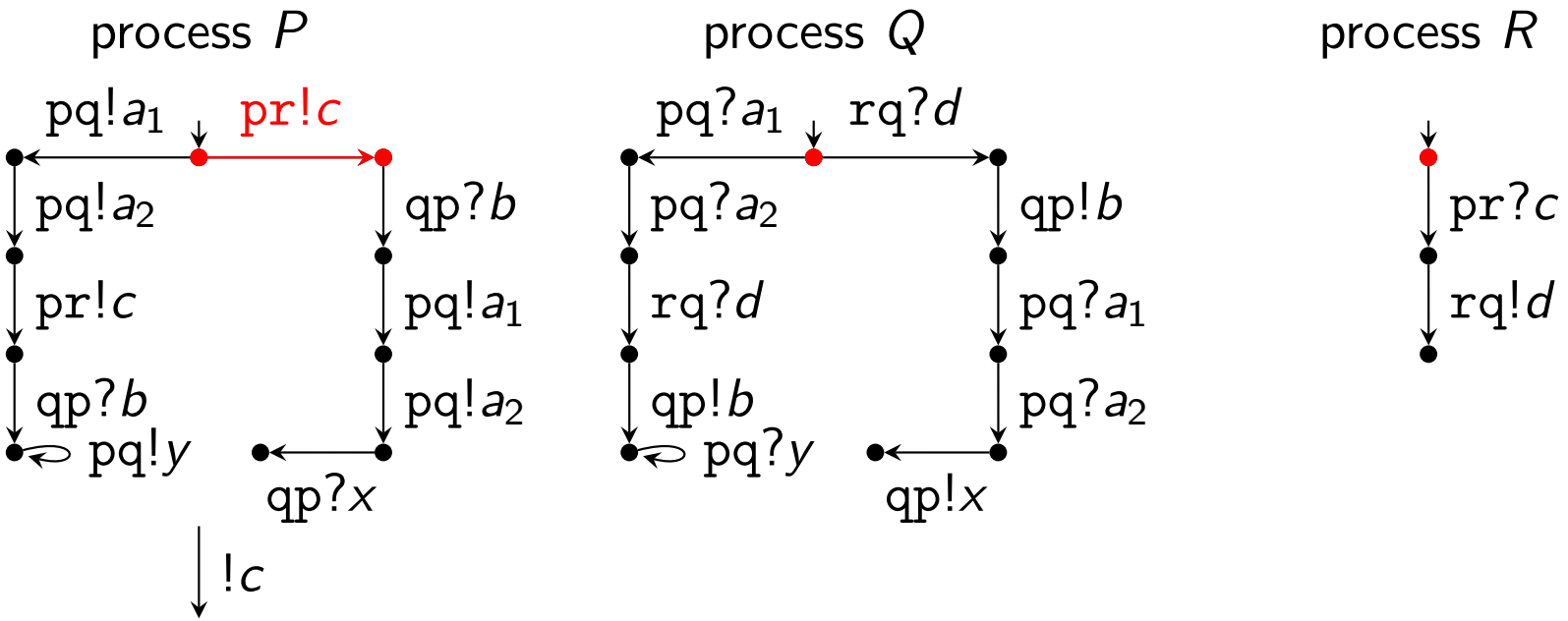


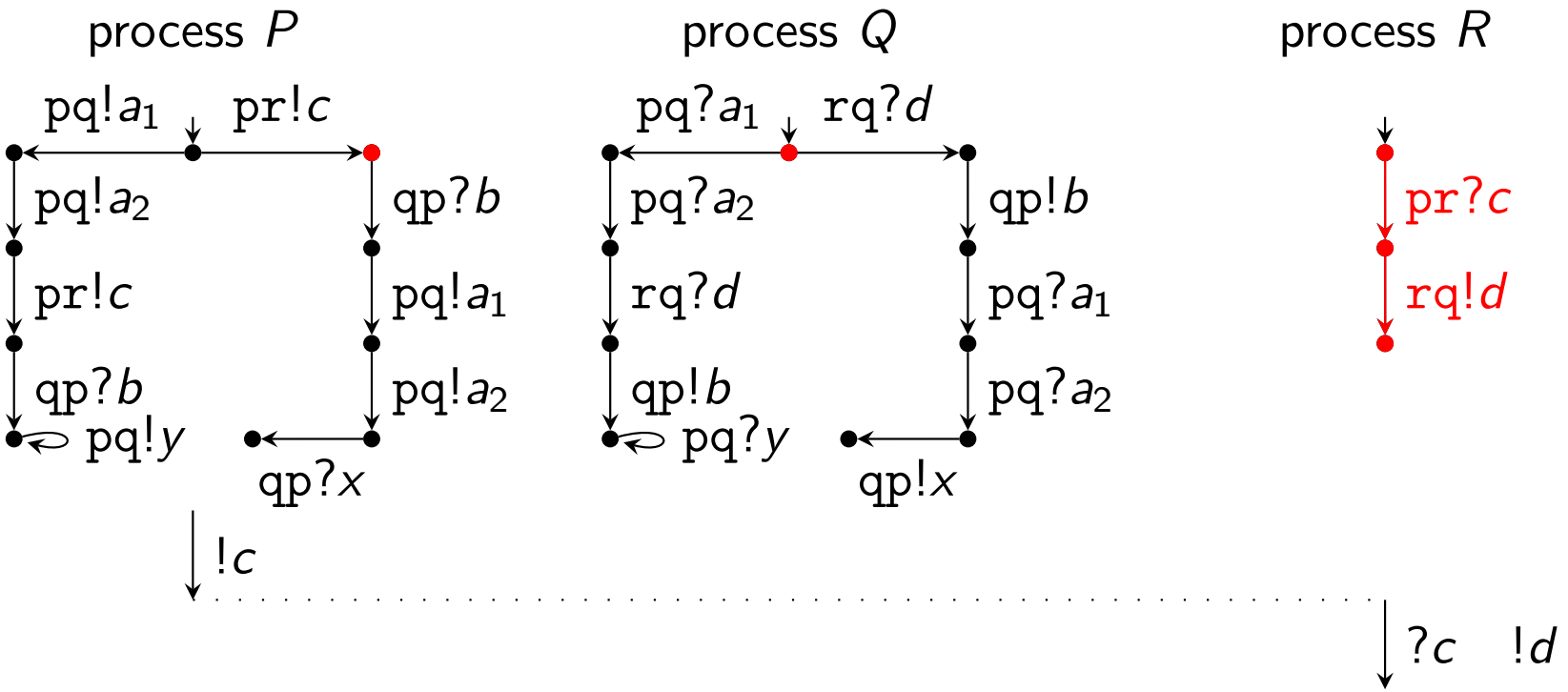
process Q

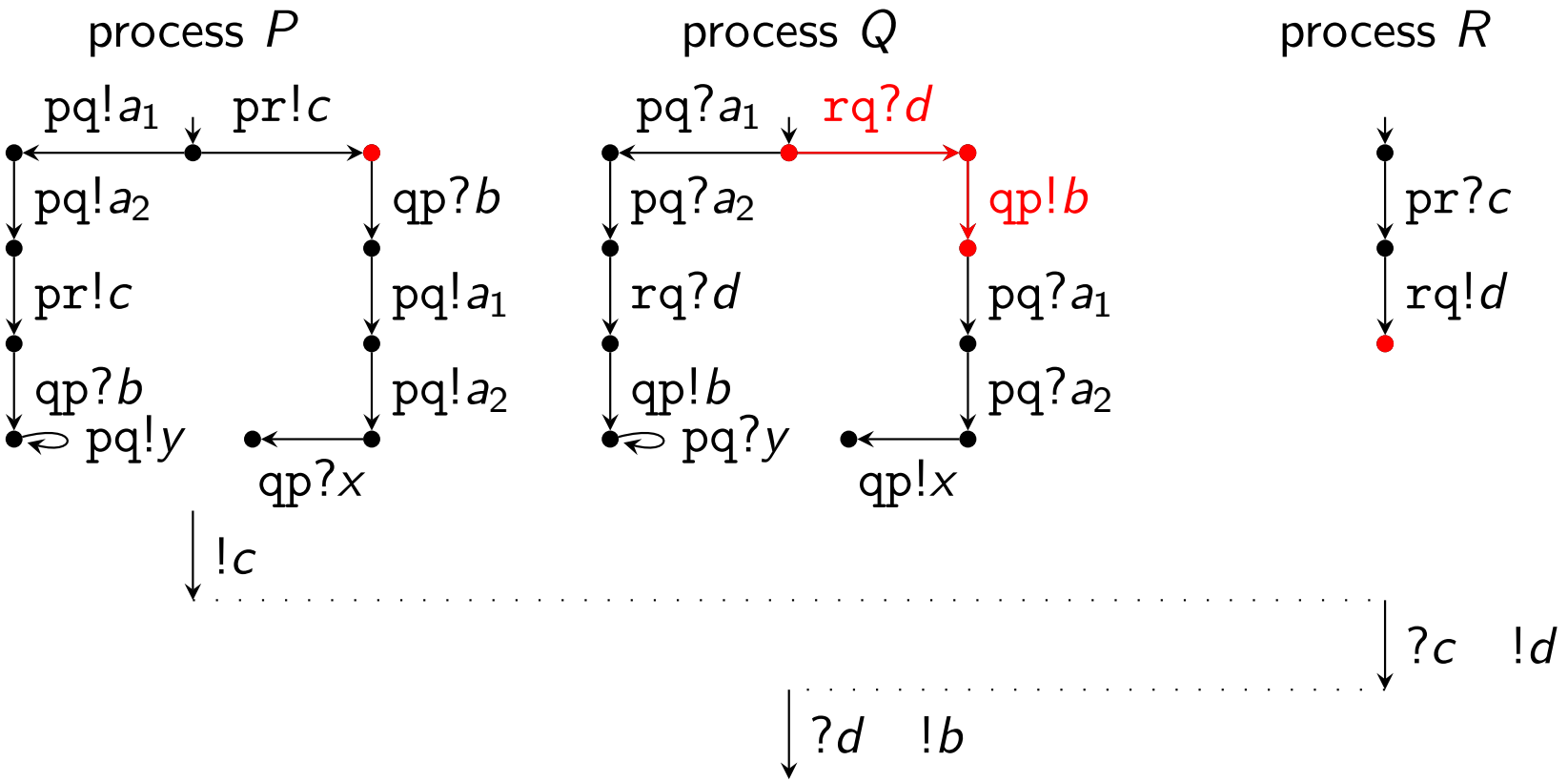


process R




process P
pq!a_1
pr!c
pq!a_2
qp?b
pr!c
pq!a_1
qp?b
pq!a_2
pq!y
qp?x
!c
process Q
pq?a_1
rq?d
pq?a_2
qp!b
rq?d
pq?a_1
qp!b
pq?a_2
pq?y
qp!x
process R
pr?c
rq!d


process P
pq!a_1
pr!c
pq!a_2
pr!c
qp?b
pq!y
qp?b
pq!a_1
pq!a_2
qp?x
!c
process Q
pq?a_1
rq?d
pq?a_2
rq?d
qp!b
qp!b
pq?y
pq?a_1
pq?a_2
qp!x
process R
pr?c
rq!d
?c
!d


process P
pq!a_1
pr!c
pq!a_2
qp?b
pr!c
pq!a_1
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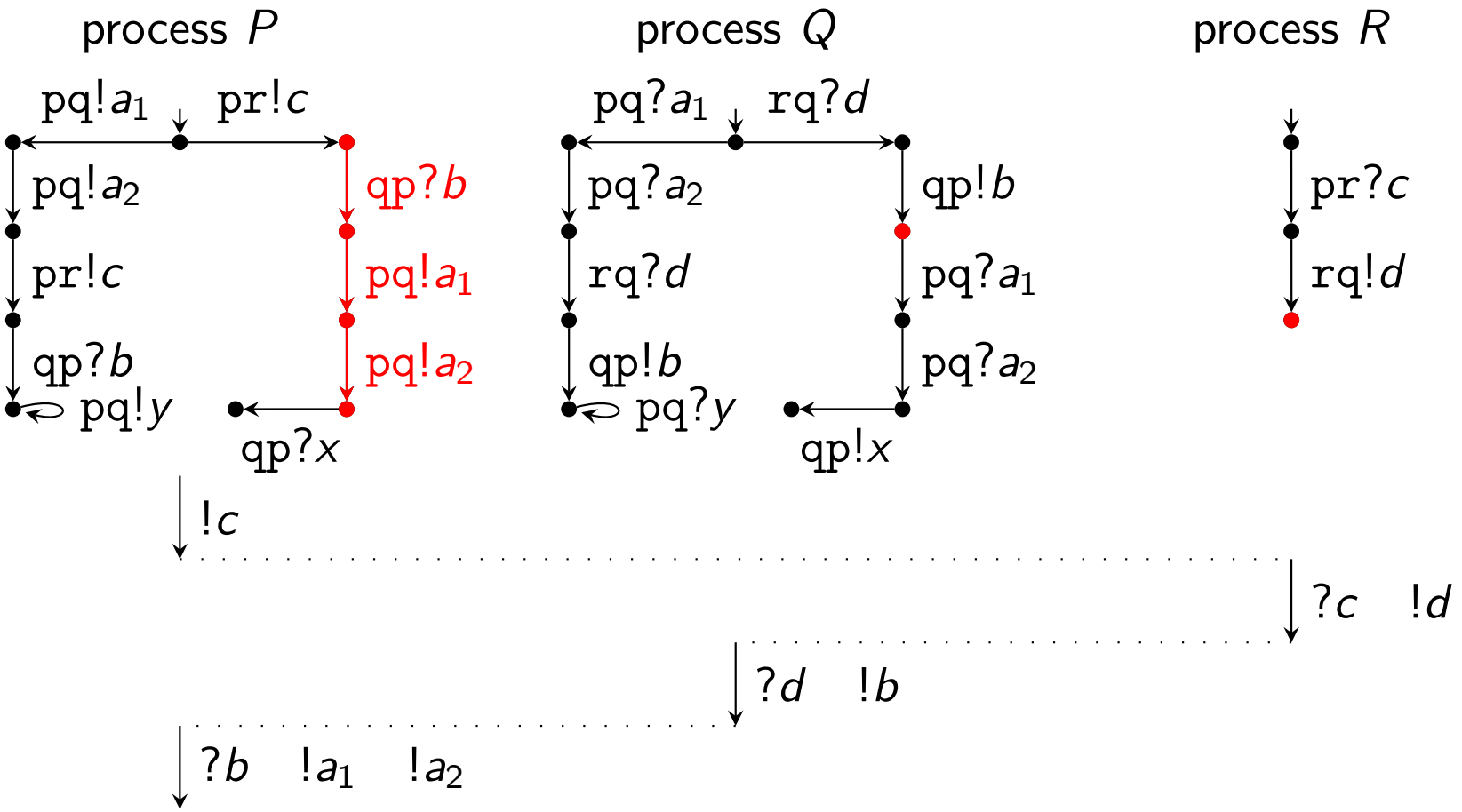
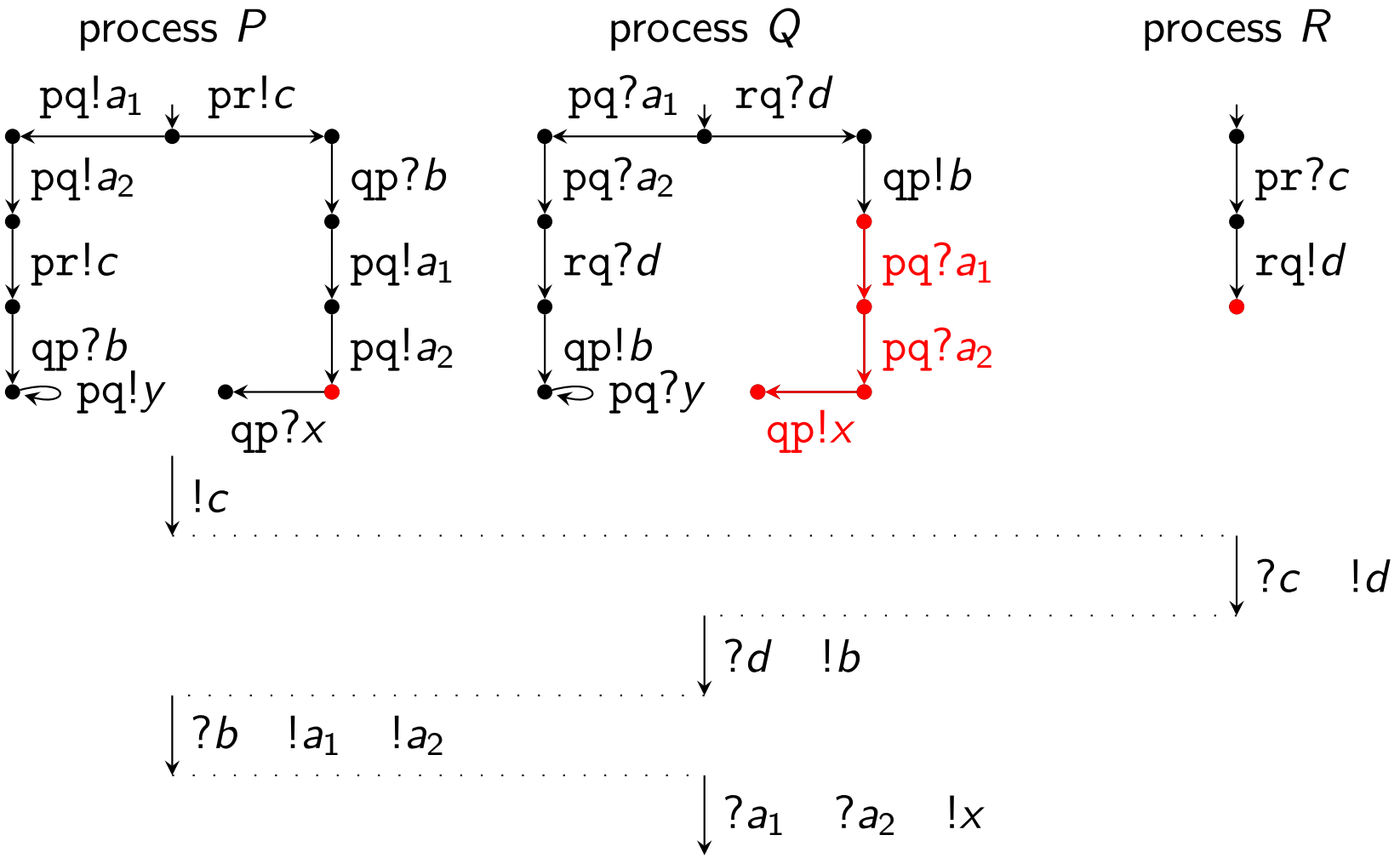
pq!a_2
pq!y
qp?x
process Q
pq?a_1
rq?d
pq?a_2
qp!b
rq?d
pq?a_1
qp!b
pq?a_2
pq?y
qp!x
process R
pr?c
rq!d
!c
?c !d
?d !b


process P
pq!a_1
pr!c
pq!a_2
pr!c
qp?b
pq!y
qp?x
qp?b
pq!a_1
pq!a_2
process Q
pq?a_1
rq?d
pq?a_2
rq?d
qp!b
pq?y
qp!x
qp!b
pq?a_1
pq?a_2
process R
pr?c
rq!d
!c
?c !d
?d !b
?b !a_1 !a_2


process P
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process R
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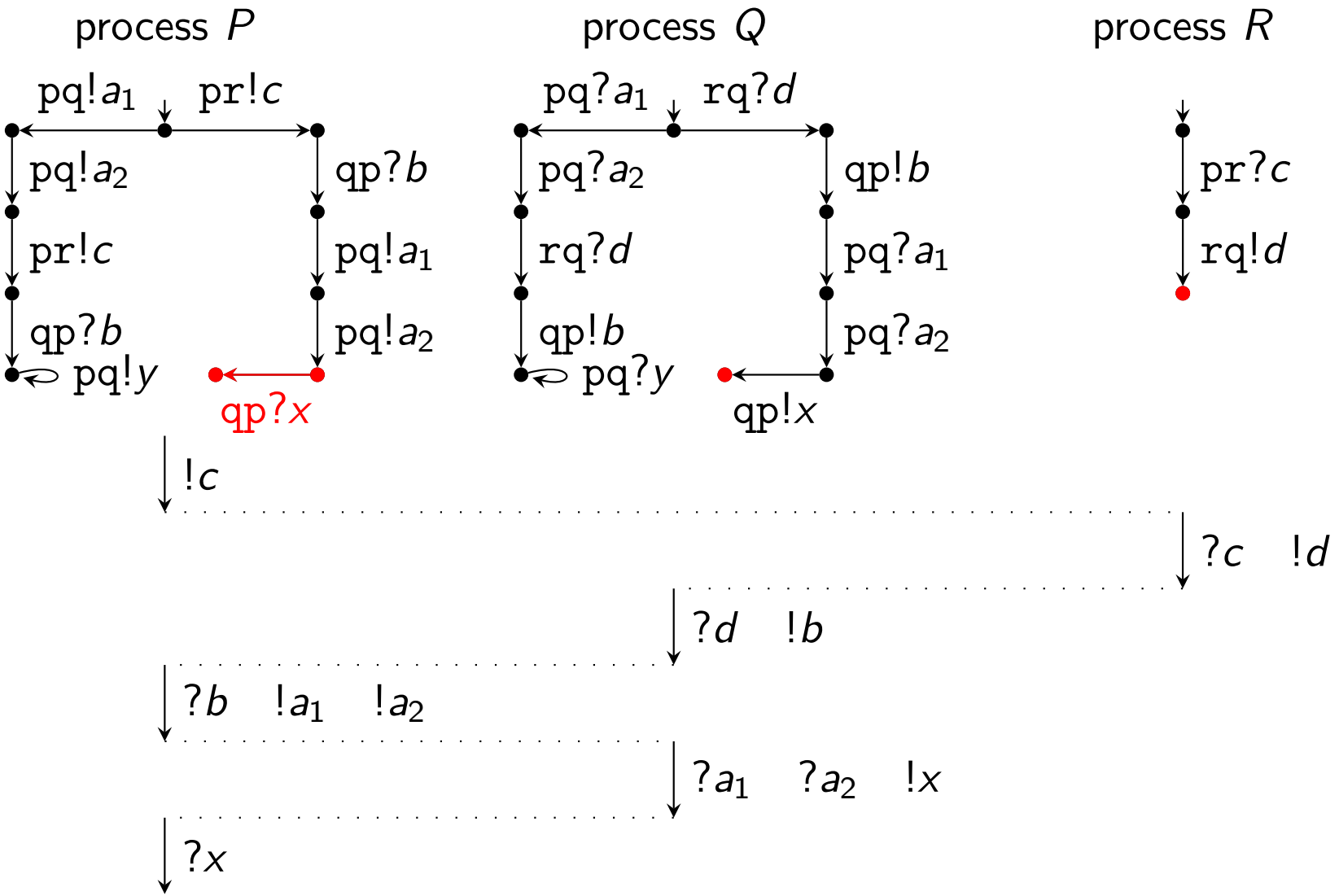
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process R
pr?c
rq!d
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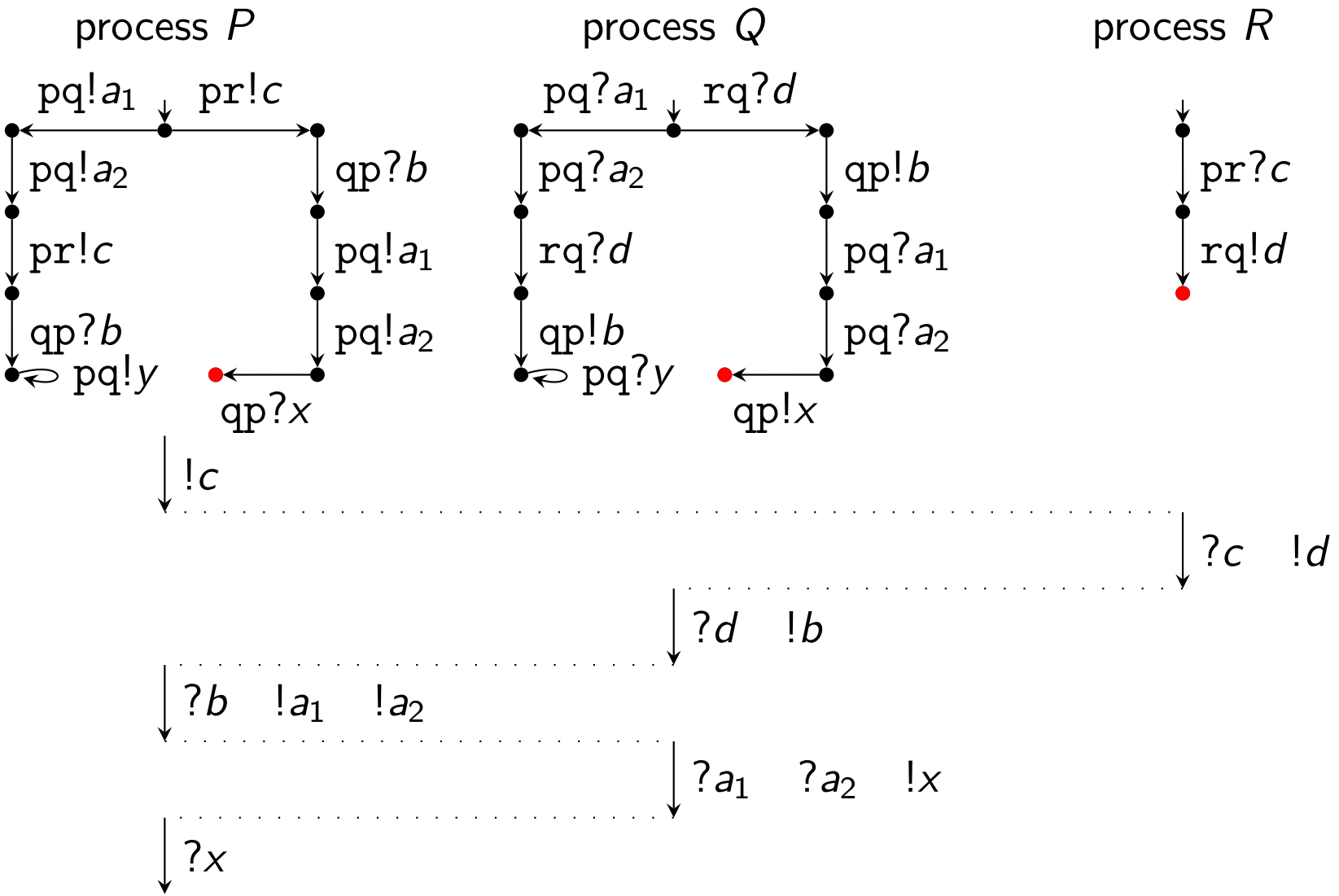
?a_1 ?a_2 !x
?x


process P
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- The reachability set is recognizable for
  - synchronous systems of CFSM
  - k-bounded systems ($k \geq 0$)
  - half-duplex systems of 2-CFSM (not for 3-CFSM).
  - lossy/insertion systems and variants with time, data and priority (but not perfect FIFO) but boundedness is still undecidable.

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  - lossy/insertion systems and variants with time, data and priority (but not perfect FIFO) but boundedness is still undecidable.
- Reachability is decidable for
  - recognizable systems
  - 1-existential bounded systems
  - flat systems.

What precisely about Flat FIFO systems (FFS) ?

Known results

- The reachability set can be effectively represented by (A, ϕ) where A is a flat automaton, ϕ Presburger formula (BH'99).
- By analysing the proof, reachability is in 2-EXPTIME.
- Control-state reachability is NP-complete (EGM'12).

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Open complexity and decidability problems

- Reachability: decidable but exact complexity unknown
- Repeated reachability ?
- (letter)-Boundedness ?
- Termination ?
- LTL, CTL^*, equivalences ?

Our contributions

- Most reachability problems are NP-complete
  - Reachability
  - Repeated reachability
  - (letter)-channel boundedness
  - Termination

Our contributions

- Most reachability problems are NP-complete
  - Reachability
  - Repeated reachability
  - (letter)-channel boundedness
  - Termination
- Flat FIFO systems are flat counters systems
  - FFS are bisimilar to FCS
  - The reachability set is semilinear (also in BH'99)
  - FFS are trace-flattable
  - LTL and CTL^* are decidable.

Outline

Two usefull lemmas

Lemma

Let $x, y \in \Sigma^+$ and $w \in \Sigma^*$.
The equation $x^\omega = wy^\omega$ holds iff $\exists z \neq \epsilon$, z primitive and $\exists x', x''$ such that $w \in x^* x'$ and $x = x'x''$ and $x''x' \in z^*$ and $y \in z^*$.

Proof.

By using Levi's Lemma. □

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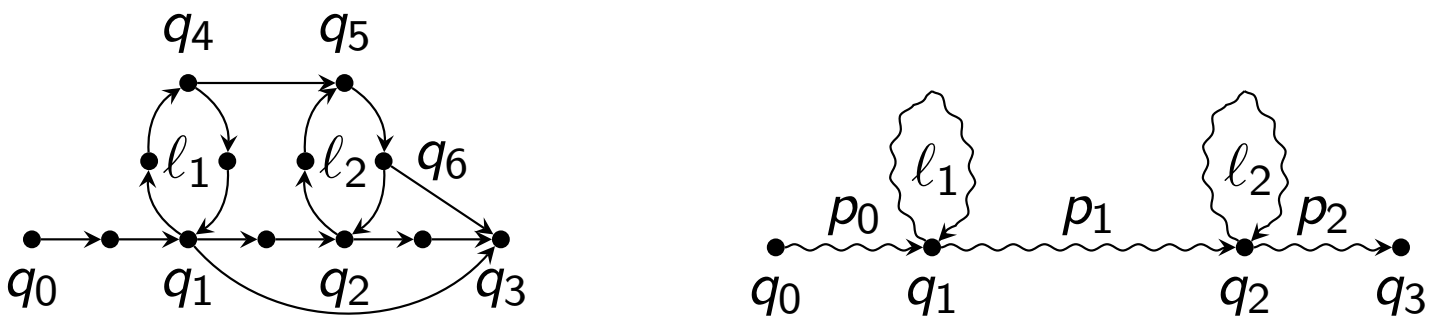
Lemma

An elementary loop labeled by σ is infinitely iterable from $(q, \mathbf{w})$ iff for every channel c, $x_c^\sigma = \epsilon$ or (σ is fireable at least once from $(q, \mathbf{w})$ and $(x_c^\sigma)^\omega = \mathbf{w}(c) \cdot (y_c^\sigma)^\omega$ and $|x_c^\sigma| \leq |y_c^\sigma|$)
where x_c^σ is the word consummed by σ from channel c.

Path Schemas



(a) Flat FIFO system

(b) Path schema denoted by $p_0(\ell_1)^* p_1 (l_2)^* p_2$

Figure: Example flat FIFO system and path schema

Reachability to Control State Reachability

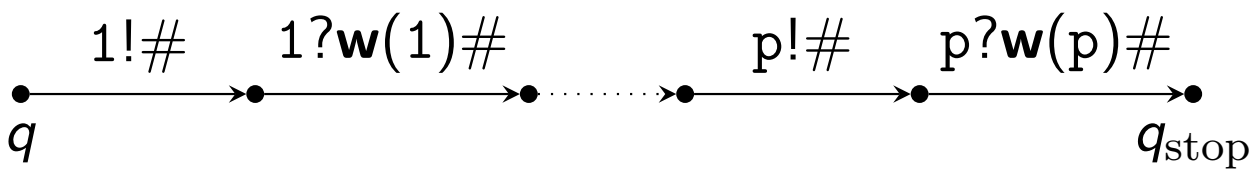
Theorem (Theorem 3, Theorem 7 in EGM'12)

Let $S = p_0(\ell_1)^* p_1 \cdots (\ell_k)^* p_k$ be a FIFO path schema.
We can compute in polynomial time an existential Presburger formula $\phi(x_1, \ldots, x_k)$ such that: there is a run $r = p_0(\ell_1)^{n_1} p_1 \cdots (\ell_k)^{n_k} p_k$ of S iff $\phi(n_1, \ldots, n_k)$ is true.
Hence control-state reachability is decidable.

Corollary

Reachability is in NP.

$(q, \mathbf{w}(1), \mathbf{w}(2), ..., \mathbf{w}(\mathrm{p}))$ is reachable iff q_{stop} is reachable.



Proposition

The repeated control state reachability problem is in NP.

Proof.

Let q be in an elementary loop labeled with σ in system S(else...).
q is infinitely repeated iff $\forall c\ [x_c^\sigma = \epsilon\ ]$ or $[\exists w\ (q, \mathbf{w}) \xrightarrow{\sigma}$ and $(x_c^\sigma)^\omega = \mathbf{w}(c) \cdot (y_c^\sigma)^\omega$ and $|x_c^\sigma| \leq |y_c^\sigma|]$ (from Lemma 2)

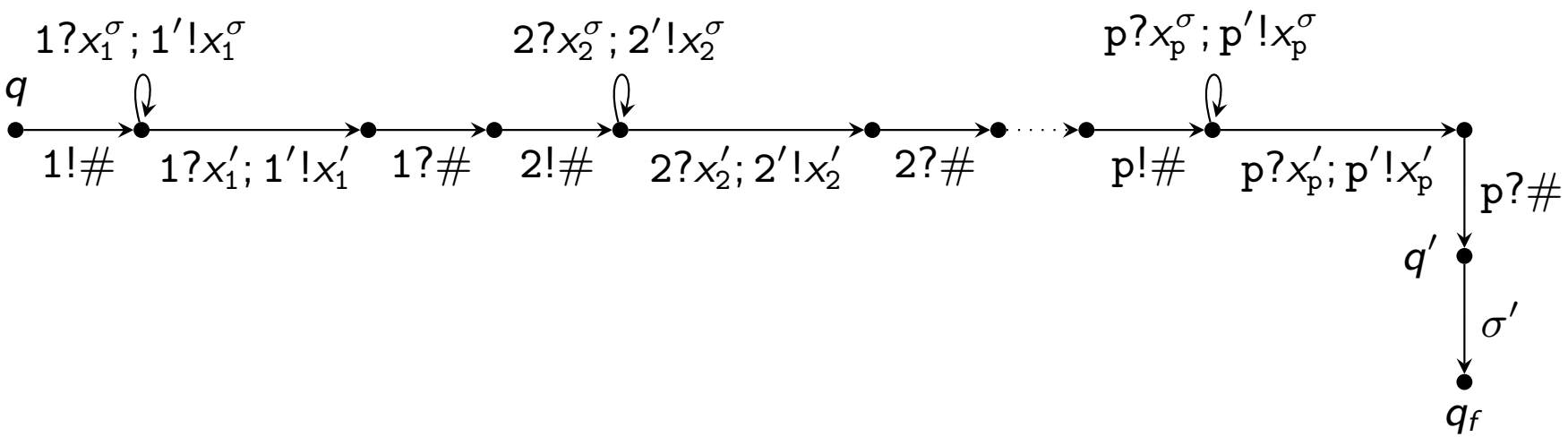
1. Verify that for every channel c, $|x_c^\sigma| \leq |y_c^\sigma|$
2. Verify $\exists (q, \mathbf{w})$ s.t. $(q, \mathbf{w}) \xrightarrow{\sigma}$ and $\forall c$ s.t. $x_c^\sigma \neq \epsilon$, $(x_c^\sigma)^\omega = \mathbf{w}(c) \cdot (y_c^\sigma)^\omega$.
3. For verifying $(x_c^\sigma)^\omega = \mathbf{w}(c) \cdot (y_c^\sigma)^\omega$ (Lemma 1), one guesses $x_c', x_c'', z_c \in M^*$ such that $x_c^\sigma = x_c' x_c''$ and $x_c'' x_c', y_c^\sigma \in z_c^*$.
4. Remark that $|x_c'|, |x_c''| \leq |x_c^\sigma|$ and $|z_c| \leq |y_c^\sigma|$
5. It remains to verify $\exists (q, \mathbf{w})$ s.t. $\forall c$, $\mathbf{w}(c) \in (x_c^\sigma)^* x_c'$ and $(q, \mathbf{w}) \xrightarrow{\sigma}$.
6. To do that, we add a channel c' for every channel c in system S.

□

Recall, we have:

- q is reached repeatedly in S iff
- $\exists \mathbf{w}(\mathrm{c})$ s.t. $\mathbf{w}(\mathrm{c}) \in (x_{\mathrm{c}}^{\sigma})^* x_{\mathrm{c}}'$ and $(q, \mathbf{w}) \xrightarrow{\sigma}$ iff
- $\exists \mathbf{w}'(\mathrm{c}')$ s.t. $\mathbf{w}'(\mathrm{c}') \in (x_{\mathrm{c}}^{\sigma})^* x_{\mathrm{c}}'$ and $(q', \mathbf{w}') \xrightarrow{\sigma'}$ iff
- q' is reachable in S' and $(q', \mathbf{w}') \xrightarrow{\sigma'}$ iff
- q_f is reachable in S'.

Hence repeated control state reachability reduces to control-state reachability.



NP Upper Bound for termination and unboundedness

Corollary

For flat FIFO systems, the non-termination and unboundedness problems are in NP.

Proof.

- Termination reduces to repeated control-state reachability since a flat system is non-terminating iff there is an infinite run r that visits at least one control state infinitely often.
- The effect of a loop ℓ labeled with σ is $\mathbf{v}_\ell \in \mathbb{Z}^F$ s.t. $\forall c \in F$ $\mathbf{v}_\ell(c) = |x_c^\sigma| - |y_c^\sigma|$.
- Unboundedness reduces to repeated control-state reachability since a flat FIFO system is unbounded iff there is at least one infinitely iterable loop ℓ with $\mathbf{v}_\ell \geq \mathbf{0}$ and $\mathbf{v}_\ell(c) \geq 1$ for some c.

□

NP Upper Bound for channel-boundedness

Proposition

The problem of checking whether a letter a is unbounded in channel c is in NP.

Proof.

In the proceedings. □

Theorem

For flat FIFO systems, reachability, repeated control-state reachability, non-termination, unboundedness, channel-unboundedness and letter-channel-unboundedness are NP-hard.

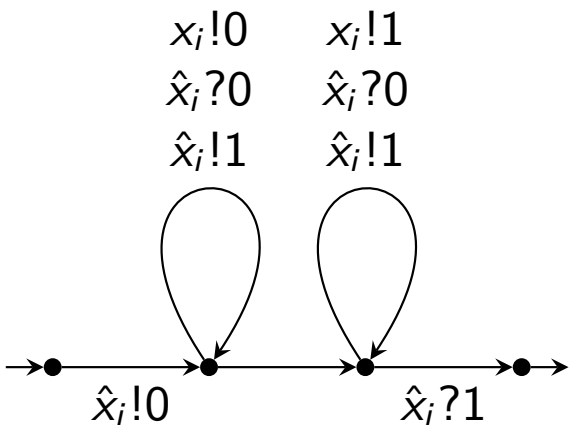
Proof.

We reduce 3-SAT to reachability. Given a 3-CNF formula $\mathrm{clause}_1 \wedge \cdots \wedge \mathrm{clause}_m$ over variables $x_1, \ldots, x_n$, we construct a flat FIFO system with $2n + m$ channels : $\{x_i, \hat{x}_i \mid i \in [1, n]\} \cup \{c_i \mid i \in [1, m]\}$.

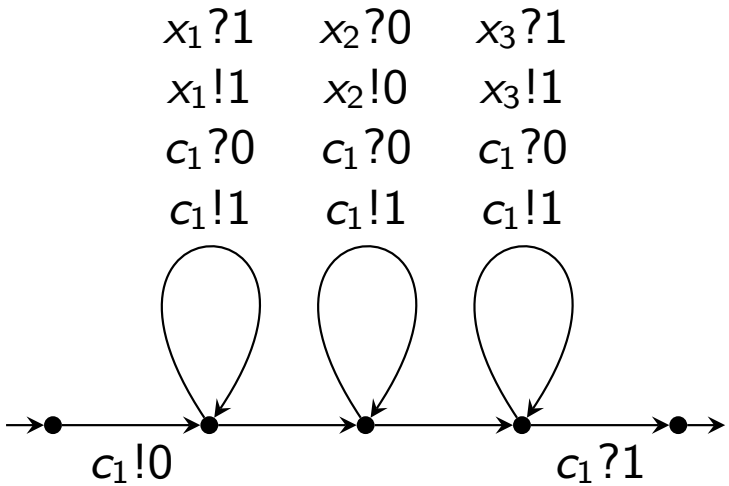
- channel x_i is used to keep a guess of the truth assignment to x_i.
- channel $\hat{x}_i$ is a "control channel" that ensures that only one guess is made.
- channel c_i is used to verify that clause_i is satisfied.

The given 3-CNF formula is satisfiable iff the last control state of the cleanup gadget for variable x_n can be reached with all channels being empty. □

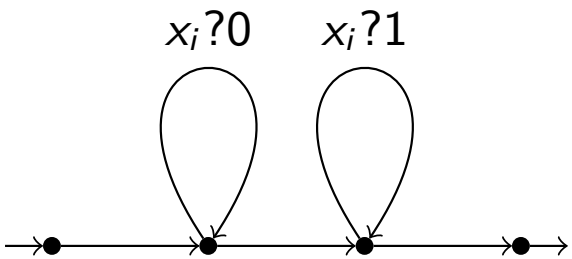
The gadget for the example clause $c_1 = x_1 \vee \neg x_2 \vee x_3$



(a) Gadget for variable x_i



(b) Gadget for clause $c_1 = x_1 \vee \neg x_2 \vee x_3$



(c) Gadget for cleaning up variable x_i

Theorem (Most properties are NP-complete)

For flat FIFO systems, the 7 reachability properties are NP-complete:

1. *reachability*
2. *repeated reachability*
3. *repeated control-state reachability*
4. *termination*
5. *boundedness*
6. *channel-boundedness*
7. *letter-channel-boundedness.*

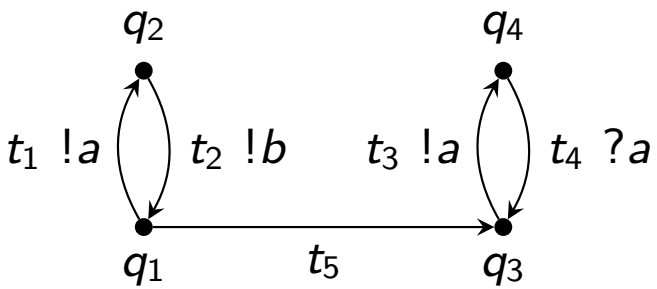
Cyclicity can be decided in linear time.

After reachability properties, model checking

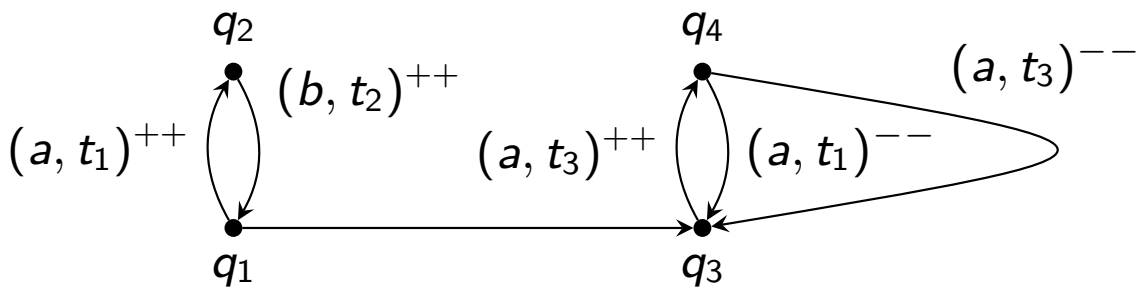
- model-checking with atomic formula $\#_c^a \geq k$
- not a consequence of the previous results (BH'99, EGM'12)
- translate a flat FIFO system into a flat counter system
- to use the existing counter systems tools

Counting abstraction system S_{count}:

- count perfectly the number of (letter $\times$ transition) sent and received
- loose the order of letters.
- $(a, t_1)^{++}$ is the incrementation of counter (a, t_1)
- $(a, t_3)^{--}$ is the decrementation of counter (a, t_3).



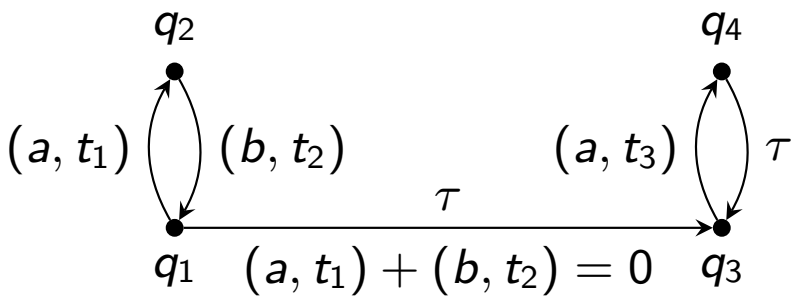
(a) Flat FIFO system



(b) Counting abstraction system S_{count}

Order system $S^{\mathsf{c}}_{\mathrm{order}}$:

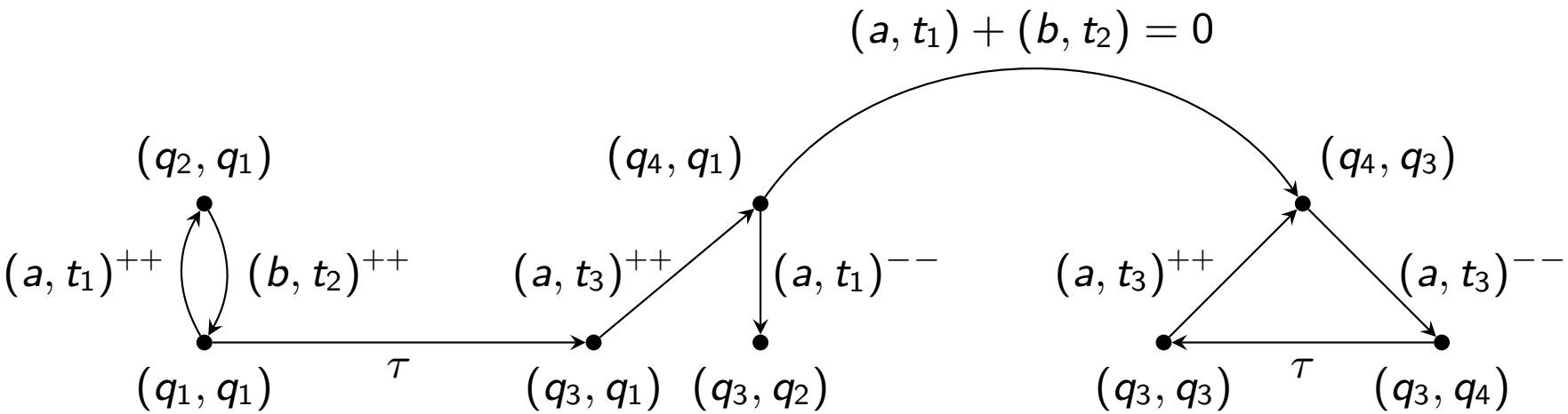
- is almost a finite automaton (it don't modify counters but makes zero-tests) that respects the FIFO policy of sent (hence received) letters.
- (b, t_2) is the label of transition from q_2 to q_1 that don't modify counters.
- its language is the sequences of sent letters : $[(a, t_1).(b, t_2)]^*.(a, t_3)^*$
- don't count so loose the number of letters.
- $(a, t_1) + (b, t_2) = 0$ means that it leaves a loop ℓ only if all letters sent by ℓ have been consummed.



(a) Order system $S^{\mathsf{c}}_{\mathrm{order}}$

Synchronized counter system

- S_{count} is synchronized with S^{c}_{order} by rendez-vous on transition labels.
- A decrementation $(a, t_1)^{--}$ in S_{count} is synchronized with the label (a, t_1) in S^{c}_{order}; this insures that receptions follows the FIFO ordering.
- Incrementations in S_{count} are not synchronized since sending is free.



(a) Synchronized counter system

Proposition

The synchronized counter system S_{sync} is (weakly) bisimilar to the flat FIFO system.

Proof.

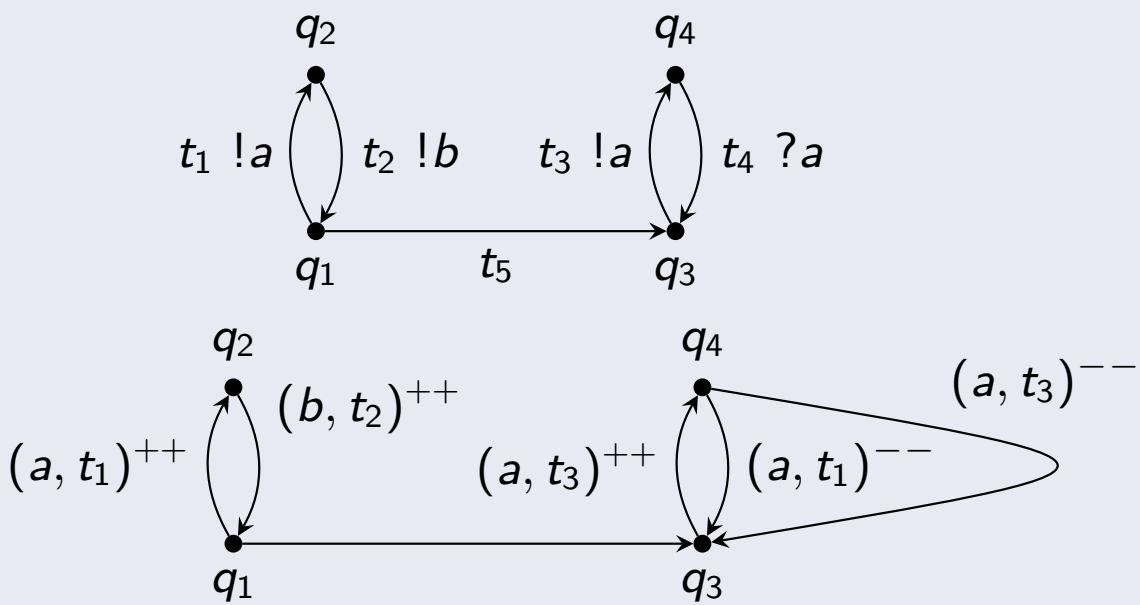
Prove the weak bisimulation by routine induction on the length of the run of S_{sync} reaching the configuration $(\overline{q}, \nu)$.
Modify the synchronized system S_{sync} to obtain a bisimulation. □

Proposition

The synchronized counter system S_{sync} is trace-flattable (hence, for example, the tool FAST will terminate).

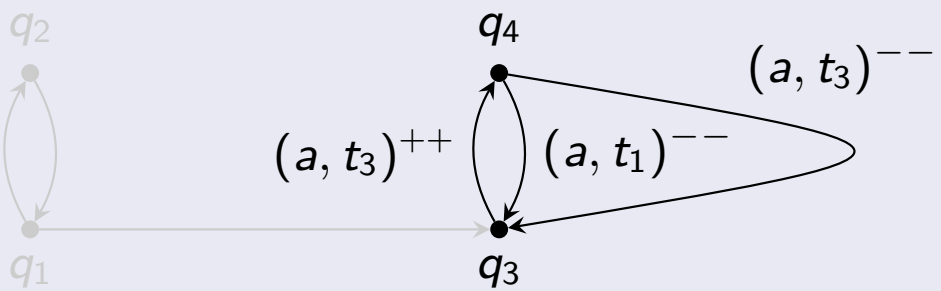
Remark

S_{count} is not flat in general.

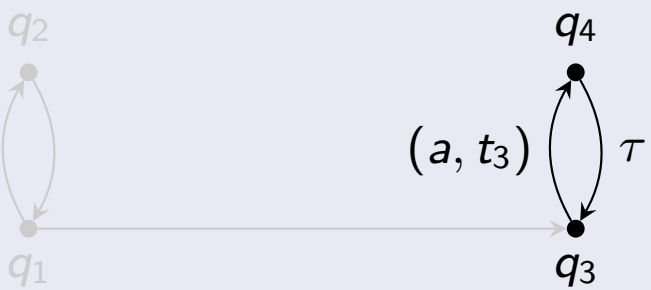


Proof.

Suppose a run is visiting states q_3, q_4 of S_{count} and states q_3, q_4 of $S^{\text{c}}_{\text{order}}$. (grey part no longer reachable).



(a) (possibly reachable) non flat S_{count}

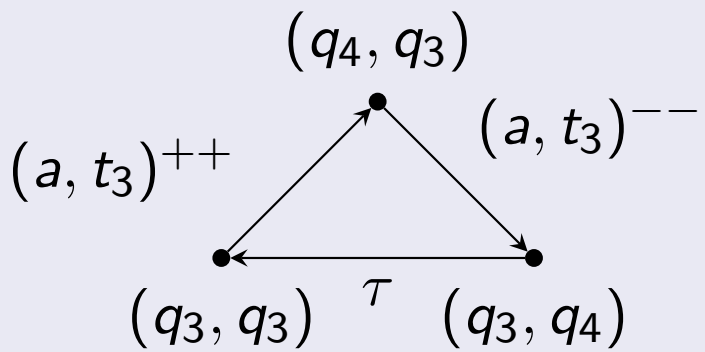


(b) (possibly reachable) $S^{\text{c}}_{\text{order}}$

Part of synchronized counter system still reachable

Proof.

The part of synchronized counter system still reachable is flat.



Theorem

LTL and CTL^* are decidable for flat FIFO systems.

Proof.

Trace-flattening preserves LTL and bisimulation preserves CTL^*. □

Open problems

Still open

- Collect case studies.
- Build and experiment a tool that flatten FIFO systems.
- Solve many open complexity problems: LTL, CTL^*, equivalences for FFS.

Info

- The paper, with complete proofs, is on HAL.
- https://hal.archives-ouvertes.fr/hal-02267453

Open post-doc

- Post-doc positions are available at LSV.
- To make theory and/or a tool for counter/FIFO systems.
- Collaborations with many researchers in LSV (ENS Paris-Saclay), LaBRI (Univ. Bordeaux), Canada, India (Chennaï, Bombay), Germany,...

Thank you