

Task D: Complexity Analysis

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COMPLEXITY FOR WQOS: “BEFORE”

- ▶ WQOs and well-founded orderings used in algorithmics: existence of a regular invariant, eventual convergence of $Pre^{\leq n}(\cdot)$ sets, etc. Also Robertson & Seymour's Theorem for polynomial-time graph decision algorithms.
- ▶ Analysing the running time of these algorithms is hard to do, especially for proving upper bounds, but also lower bounds. Our standard tools do not apply there.
- ▶ What is needed are upper bounds on the length of bad sequences. These bounds are necessarily parameterized by the processes that construct the sequences.
- ▶ Early solutions (McAloon et al.) only for VAS, not well-known, not been much used.

COMPLEXITY FOR WQOS: “A NEW FRAMEWORK”

- ▶ Over the last 10 year, we (Schmitz+phs+..) developed a more generic and more complete approach for proving upper bounds.
- ▶ VAS, i.e., \mathbb{N}^k are a special case: we can deal with more complex wqos. Gist of the idea is relating the length of bad sequences to the computation of “maximal order types” and their connection to the fast-growing hierarchy.
- ▶ Comes with natural complexity classes $(\mathbb{F}_\alpha)_\alpha$, complete problems, reductions, etc. See [Complexity hierarchies beyond Elementary](#). ACM Trans. Computation Theory, 8(1), 2016.
- ▶ Has proved very useful for several new results, is being adopted by the community.
- ▶ **We can use this framework for assessing complexity within the BRAVAS main goals and we can develop the framework as part of BRAVAS project.**

SUBTASK 1: “UPPER BOUNDS FOR MORE WQOS”

- ▶ Going from $o(X, \leq)$ to Length Function Theorems require some work (feel like combinatorics) that has not yet been always done: only for elementary wqos.
- ▶ In some cases $o(X, \leq)$ is not known precisely: powersets, trees and graphs, exotic wqos.

SUBTASK 2: “COMPLEXITY FOR IDEAL-BASED ALGORITHMS”

- ▶ In principle our upper bounds techniques based on Length Functions should apply but algorithms must be described at a less abstract level.
- ▶ This question is relevant for KLM-based algorithms