Open Boundedness and Coverability Problems for Pushdown Vector Addition Systems

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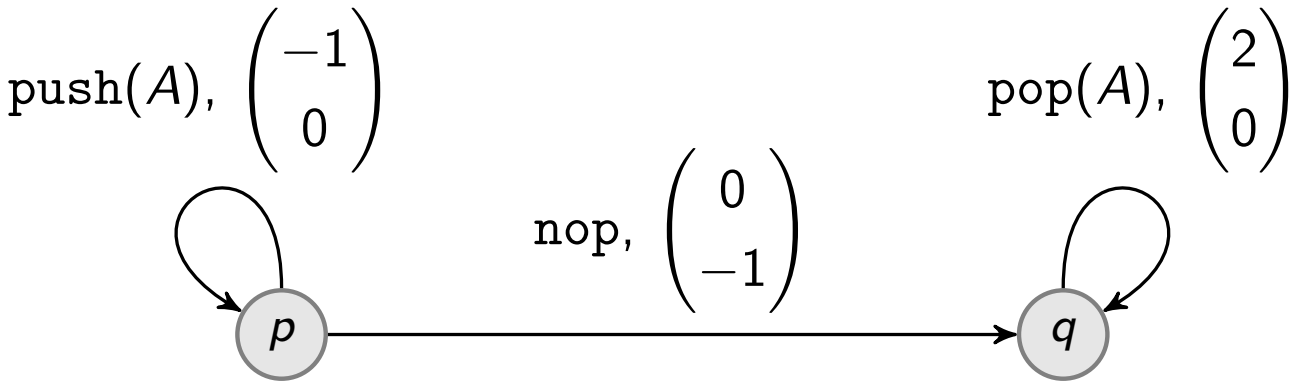
Pushdown Vector Addition Systems

. . . are products of VAS with pushdown automata



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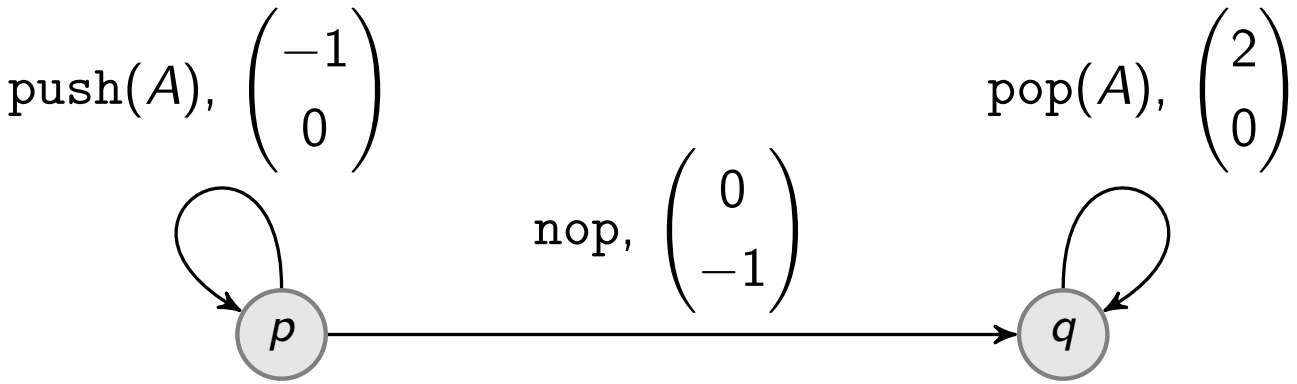

$p, \bot, \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

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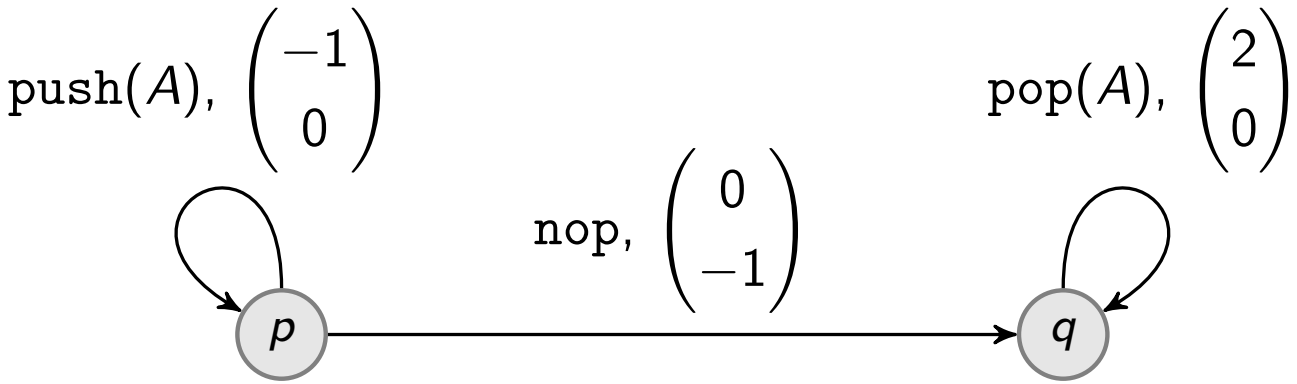

$$p, \bot, \begin{pmatrix} 2 \\ 1 \end{pmatrix} \longrightarrow\longrightarrow p, AA\bot, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

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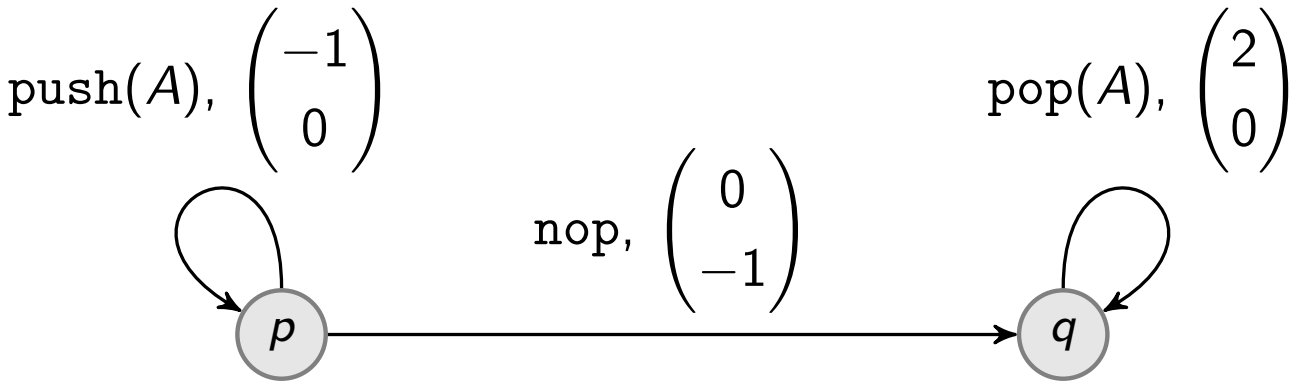

$$p, \bot, \begin{pmatrix}2\\1\end{pmatrix} \longrightarrow\longrightarrow p, AA\bot, \begin{pmatrix}0\\1\end{pmatrix} \longrightarrow q, AA\bot, \begin{pmatrix}0\\0\end{pmatrix}$$

Pushdown Vector Addition Systems

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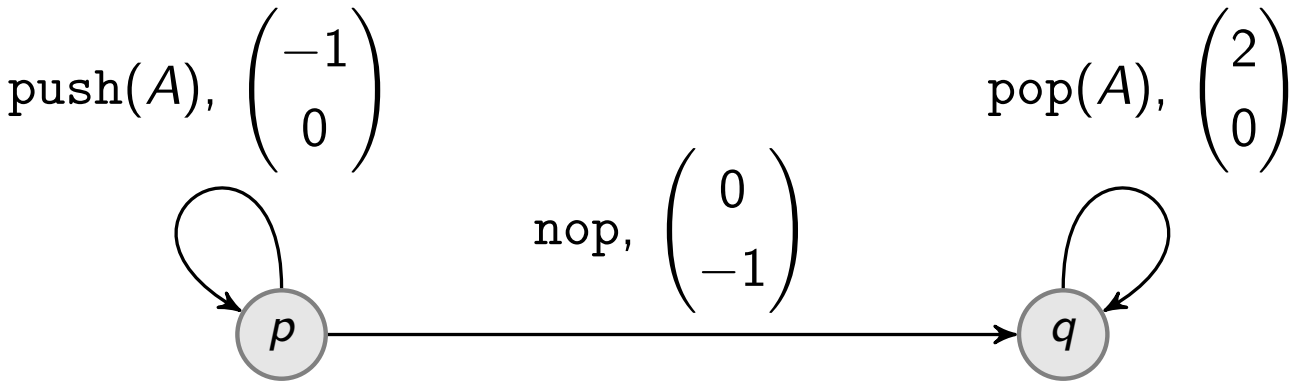


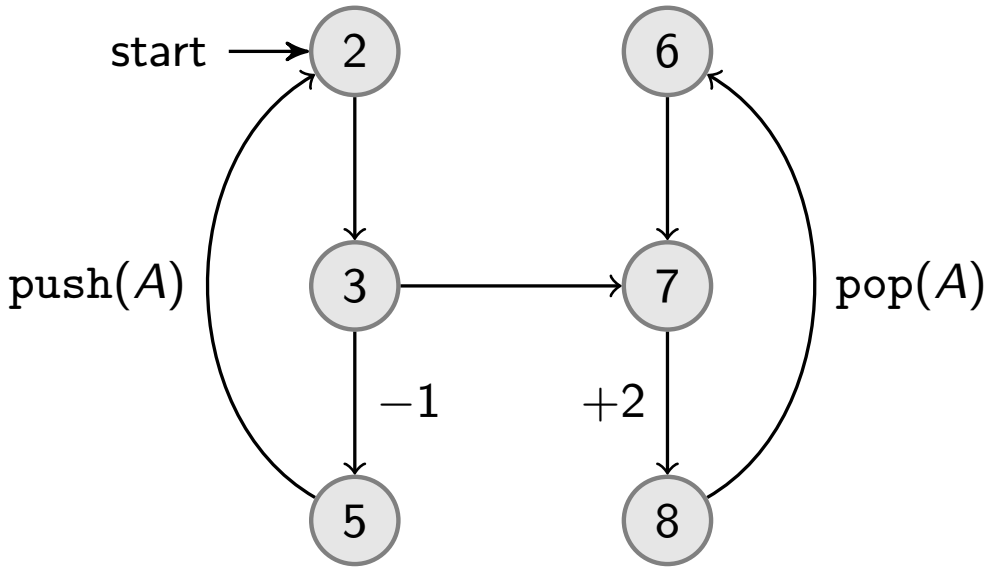
$$p, \bot, \begin{pmatrix}2\\1\end{pmatrix} \longrightarrow\longrightarrow p, AA\bot, \begin{pmatrix}0\\1\end{pmatrix} \longrightarrow q, AA\bot, \begin{pmatrix}0\\0\end{pmatrix} \longrightarrow\longrightarrow q, \bot, \begin{pmatrix}4\\0\end{pmatrix}$$

Pushdown Vector Addition Systems

. . . are products of VAS with pushdown automata

➪ They can for example model recursive programs with variables over $\mathbb{N}$

```
1: x ← n
2: procedure DoubleX
3:     if (⋆ ∧ x > 0) then
4:         x ← (x − 1)
5:         DoubleX
6:     end if
7:     x ← (x + 2)
8: end procedure
```



Pushdown Vector Addition Systems

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➪ They can for example model recursive programs with variables over $\mathbb{N}$

➪ They can also model concurrent systems with (limited) recursion

- One recursive server + unboundedly many finite-state clients

➪ Is the model too powerful?

Brief non-Exhaustive State of the Art

- Reachability: does $(p, \varepsilon, \boldsymbol{u}) \xrightarrow{*} (q, \varepsilon, \boldsymbol{v})$?
- Coverability: does there exist $\boldsymbol{v}' \geq \boldsymbol{v}$ with $(p, \varepsilon, \boldsymbol{u}) \xrightarrow{*} (q, \varepsilon, \boldsymbol{v}')$?
- Boundedness: is $\{(q, \sigma, \boldsymbol{v}) \mid (p, \varepsilon, \boldsymbol{u}) \xrightarrow{*} (q, \sigma, \boldsymbol{v})\}$ finite ?

	Boundedness	Coverability	Reachability
VAS	ExpSpace-c[1]	ExpSpace-c[1]	Decidable[2]
+ full counter	Decidable[4]	Decidable[3]	
+ stack	Decidable[6]	Tower-h[5], ?	

[1] Lipton 1976 ; Rackoff 1978

[2] Mayr 1981 ; Kosaraju 1982 ; Leroux, Schmitz 2015

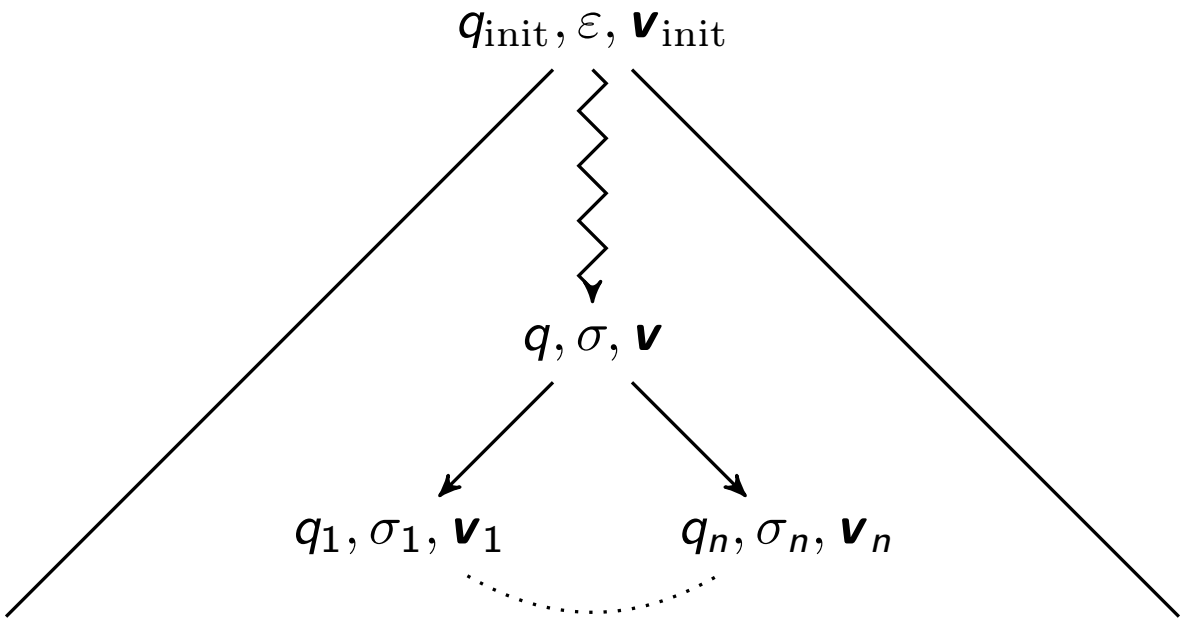
[3] Reinhardt 2008

[4] Finkel, Sangnier 2010

[5] Lazić 2012

[6] Leroux, Praveen, Sutre 2014

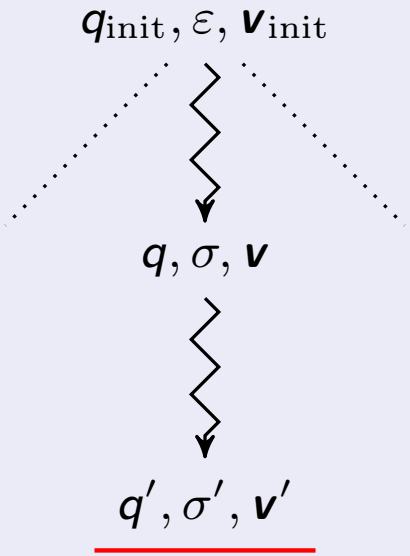
Reachability Tree of a Pushdown VAS



- Exhaustive and enumerative forward exploration from $(q_{\text{init}}, \varepsilon, \boldsymbol{v}_{\text{init}})$
- Potentially infinite, need to truncate

Tentative Simulation-Based Truncation for Pushdown VAS

Truncation Rule



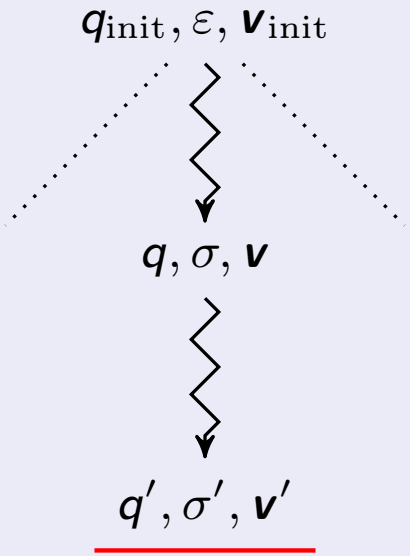
if $q = q'$, $\boldsymbol{v} \leq \boldsymbol{v}'$ and $\sigma \leq_{\text{prefix}} \sigma'$

➪ No loss of information to decide boundedness

But...

Tentative Simulation-Based Truncation for Pushdown VAS

Truncation Rule



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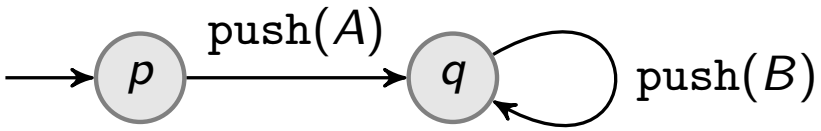


$(p, \bot)$

$(q, A\bot)$

$(q, BA\bot)$

$(q, BBA\bot)$

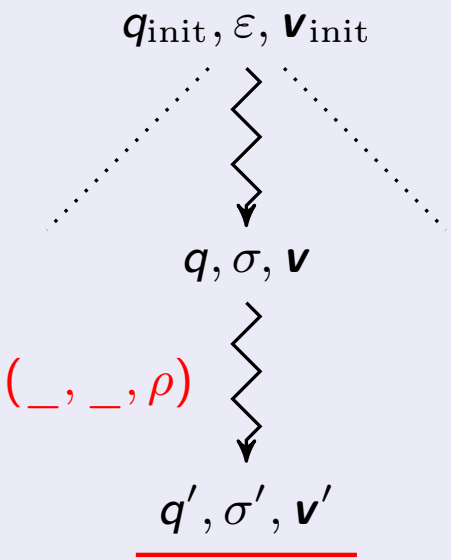
⇨ No loss of information to decide boundedness

But...

The reduced reachability tree may be infinite!

Reduced Reachability Tree for Pushdown VAS

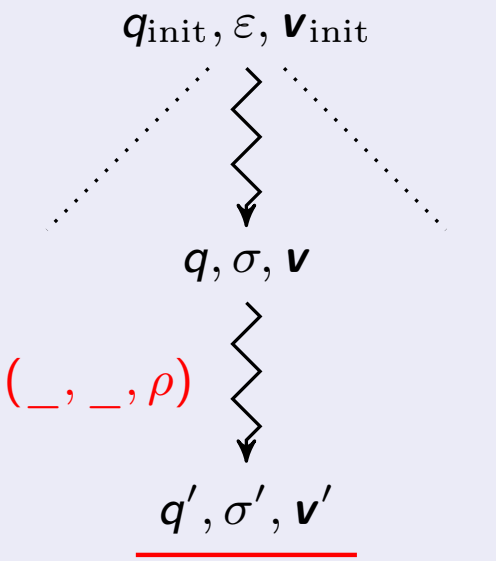
Truncation Rule



$$\text{if } \begin{cases} q = q' \text{ and } \boldsymbol{v} \leq \boldsymbol{v}' \\ \sigma \leq_{\text{suffix}} \rho \text{ for all } \rho \end{cases}$$

Reduced Reachability Tree for Pushdown VAS

Truncation Rule



$$\text{if } \begin{cases} q = q' \text{ and } \boldsymbol{v} \leq \boldsymbol{v}' \\ \sigma \leq_{\text{suffix}} \rho \text{ for all } \rho \end{cases}$$

The reduced reachability tree

- is computable and finite
- contains enough information to decide boundedness
- has at most an hyper-Ackermannian number of nodes (the bound is tight)

Theorem ([Leroux, Praveen, S. 2014])

Boundedness is decidable for pushdown VAS

Open Boundedness Problems

Boundedness Problem for Pushdown VAS

- Lower bound: tower of exponentials ($\mathbf{F}_3$) from [Lazić 2012]
- Upper bound: hyper-Ackermann ($\mathbf{F}_{\omega^\omega}$)

Refinements: stack-boundedness and counters-boundedness

Counters-Boundedness Problem for Pushdown VAS

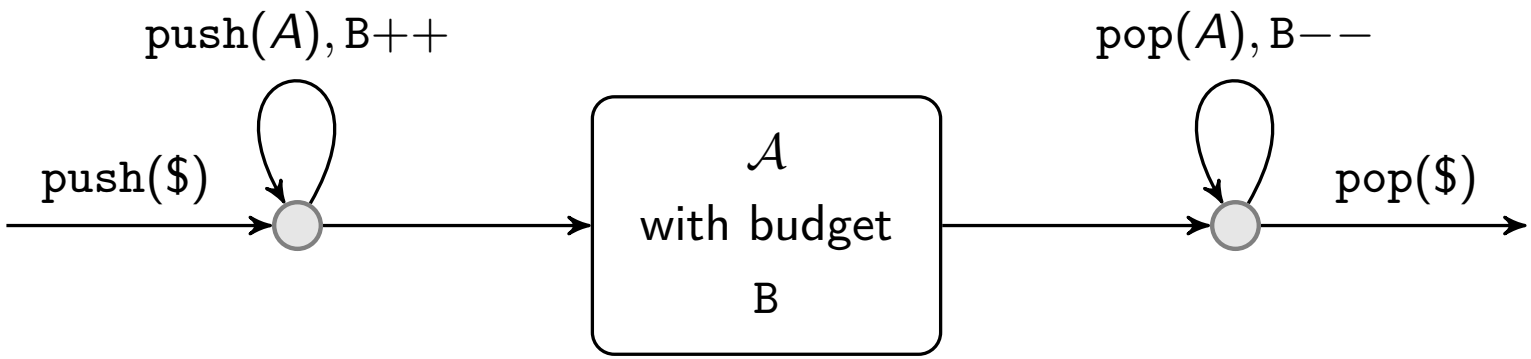
- Decidability: open
- Dim. one: NP-hard, ExpTime-easy [Leroux, Sutre, Totzke 2015]

Location-specific boundedness is at least as hard as reachability

Observation ([Lazić 2012])

Reachability in dimension d reduces to Coverability in dimension $d+1$.

Proof. Budget construction. Use the stack to test the budget for zero. Add a new counter $\mathtt{B}$ and two new stack symbols $A, \$$.



$(q^{\mathcal{A}}_{\text{init}}, \varepsilon, \mathbf{0}) \xrightarrow{*} (q^{\mathcal{A}}_{\text{final}}, \varepsilon, \mathbf{0})$ iff $(q^{\mathcal{A}'}_{\text{init}}, \varepsilon, \mathbf{0}, 0) \xrightarrow{*} (q^{\mathcal{A}'}_{\text{final}}, \varepsilon, \_, \_)$ □

Coverability versus Reachability in Pushdown VAS

Observation ([Lazić 2012])

Reachability in dimension d reduces to Coverability in dimension $d + 1$.

$$\mathit{Reach}(0) \sqsubseteq \mathit{Cover}(1) \sqsubseteq \mathit{Reach}(1) \sqsubseteq \mathit{Cover}(2) \sqsubseteq \cdots$$

Coverability versus Reachability in Pushdown VAS

Observation ([Lazić 2012])

Reachability in dimension d reduces to Coverability in dimension $d + 1$.

$$\mathit{Reach}(0) \sqsubseteq \mathit{Cover}(1) \sqsupseteq \mathit{Reach}(1) \sqsubseteq \mathit{Cover}(2) \sqsubseteq \cdots$$

Theorem ([Leroux, Sutre, Totzke 2015])

Coverability for 1*-dimensional pushdown VAS is decidable.*

Open Coverability/Reachability Problems

Coverability Problem for Pushdown VAS

- Lower bound: tower of exponentials ($\mathbf{F}_3$) from [Lazić 2012]
- Decidability: open

Coverability for Pushdown VAS in Dimension One

- PSPACE-hard by reduction from bounded OCA [Lazić et al.]
- EXPSPACE-easy [Leroux, Sutre, Totzke 2015]

Reachability is open even in dimension one